

ANALYSIS OF THREE-DIMENSIONAL VISCOUS FLOW IN A SUPERSONIC THROUGHFLOW FAN

by

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Abstract

A three-dimensional Navier-Stokes code has been developed for analysis of turbomachinery blade rows and other internal flows. The Navier-Stokes equations are written in a Cartesian coordinate system rotating about the x -axis, and then mapped to a general body-fitted coordinate system. Streamwise viscous terms are neglected using the thin-layer assumption, and turbulence effects are modelled using the Baldwin-Lomax turbulence model. The equations are discretized using finite differences on stacked C-type grids and are solved using a multistage Runge-Kutta algorithm with a spatially-varying time step and implicit residual smoothing.

Calculations were made of the flow around a supersonic throughflow fan blade. The fan was designed at NASA Lewis Research Center as a key component in a supersonic cruise engine. It was designed to produce a total pressure ratio of 2.7 at an axial Mach number of 2.0. The midspan section of the blade is being tested in a supersonic linear cascade at Virginia Polytechnic Institute and will be tested in a rotating rig at Lewis in the near future. Comparisons between earlier quasi-3-D calculations and the VPI data show excellent agreement between shock locations and wake traverses.

The 3-D calculations were done on a $129 \times 29 \times 33$ grid and took 50 minutes of cpu time on a Cray X-MP. Comparisons with the quasi-3-D results show minor differences in loading due to 3-D effects. Particle traces show nearly 2-D flows near the pressure surface, but large secondary flows within the suction surface boundary layer. The horseshoe vortex ahead of the leading edge is clearly seen.

References

1. Chima, R. V., and Yokota, J. W. "Numerical Analysis of Three- Dimensional Viscous Internal Flows," NASA TM-100878, July, 1988.
2. Schmidt, J. F., Moore, R. D., and Wood, J. R. "Supersonic Throughflow Fan Design," NASA TM-88908, AIAA-87-1746, June, 1987.

RVC3D (ROTOR VISCOUS CODE 3-D)

BY R. V. CHIMA

DESCRIPTION

- EULER OR NAVIER-STOKES ANALYSIS FOR STEADY 3-D FLOWS IN TURBOMACHINERY BLADE PASSAGES

FEATURES

- STACKED C-TYPE GRIDS FOR AXIAL OR CENTRIFUGAL MACHINES
- CARTESIAN FORMULATION ROTATING ABOUT X-AXIS
RECTANGULAR OR ANNULAR GEOMETRIES
- SOLVES NAVIER-STOKES EQUATIONS IN FINITE-DIFFERENCE FORM
THIN-LAYER FORMULATION NEGLECTS STREAMWISE VISCOUS TERMS
RETAINS HUB-TO-TIP & BLADE-TO-BLADE VISCOUS TERMS
BALDWIN-LOMAX TURBULENCE MODEL
- EXPLICIT 4-STAGE RUNGE-KUTTA TIME-MARCHING SCHEME
VARIABLE $\Delta t_{i,j}$ & IMPLICIT RESIDUAL SMOOTHING
HIGHLY VECTORIZED FOR CRAY X-MP

RESULTS

- SUPERSONIC THROUGHFLOW FAN

GOVERNING EQUATIONS

$$\partial_t q + J[\partial_\xi \hat{E} + \partial_\eta \hat{F} + \partial_\zeta \hat{G} - Re^{-1}(\partial_\eta \hat{F}_V + \partial_\zeta \hat{G}_V)] = H$$

WHERE:

$$\begin{aligned} q &= [\rho, \quad \rho u, \quad \rho v, \quad \rho w, \quad e]^T \\ H &= [0, \quad 0, \quad -\Omega \rho w, \quad \Omega \rho v, \quad 0]^T \\ \hat{E} &= J^{-1} [\rho U', \quad \rho u U' + \xi_x p, \quad \rho v U' + \xi_y p, \quad \rho w U' + \xi_z p, \quad e U' + p U]^T \\ \hat{F} &= J^{-1} [\rho V', \quad \rho u V' + \eta_x p, \quad \rho v V' + \eta_y p, \quad \rho w V' + \eta_z p, \quad e V' + p V]^T \\ \hat{G} &= J^{-1} [\rho W', \quad \rho u W' + \zeta_x p, \quad \rho v W' + \zeta_y p, \quad \rho w W' + \zeta_z p, \quad e W' + p W]^T \end{aligned}$$

RELATIVE VELOCITIES:

$$u' = u$$

$$v = v - \Omega z$$

$$w' = w + \Omega y$$

RELATIVE CONTRAVARIANT VELOCITIES:

$$U' = \xi_x u + \xi_y v' + \xi_z w'$$

$$V' = \eta_x u + \eta_y v' + \eta_z w'$$

$$W' = \zeta_x u + \zeta_y v' + \zeta_z w'$$

ENERGY AND STATIC PRESSURE:

$$e = \rho [C_v T + (u^2 + v^2 + w^2)/2]$$

$$p = (\gamma - 1) [e - \rho(u^2 + v^2 + w^2)/2]$$

MULTISTAGE RUNGE-KUTTA ALGORITHM

GOVERNING EQUATIONS

$$\partial_t q = -J [R_I - (R_V + D)]$$

R_I = INVISCID RESIDUAL

R_V = VISCOUS RESIDUAL

D = ARTIFICIAL DISSIPATION TERM

MULTISTAGE SCHEME

$$q_0 = q_n$$

$$q_1 = q_0 - \alpha_1 J \Delta t [R_I q_0 - (R_V + D) q_0]$$

\vdots

$$q_k = q_0 - \alpha_k J \Delta t [R_I q_{k-1} - (R_V + D) q_0]$$

$$q_{n+1} = q_k$$

R_V & D EVALUATED AT FIRST STAGE ONLY

ARTIFICIAL DISSIPATION

NONCONSERVATIVE VERSION OF JAMESON FORMULATION

$$Dq = (D_\xi + D_\eta + D_\zeta) q$$

ξ -DIRECTION OPERATOR

$$D_\xi q = C_\xi (V_2 q_{\xi\xi} - V_4 q_{\xi\xi\xi\xi})$$

WHERE:

$$C_\xi = \frac{1}{J} \left(\frac{1}{\Delta t_\eta} + \frac{1}{\Delta t_\zeta} \right) \simeq \frac{a}{J} \left(\frac{1}{\Delta s_\eta} + \frac{1}{\Delta s_\zeta} \right)$$

$$V_2 = \mu_2 \max(\nu_{i+1}, \nu_i, \nu_{i-1})$$

$$V_4 = \max(0, \mu_4 - V_2)$$

AND

$$\nu_{i,j} = \frac{|P_{i+1,j} - 2P_{i,j} + P_{i-1,j}|}{|P_{i+1,j} + 2P_{i,j} + P_{i-1,j}|}$$

$$\mu_2 = O(1)$$

$$\mu_4 = O\left(\frac{1}{16}\right)$$

IMPLICIT RESIDUAL SMOOTHING

USE A TIME STEP GREATER THAN THE STABILITY LIMIT
MAINTAIN STABILITY BY SMOOTHING THE RESIDUAL IMPLICITLY

$$(1 - \epsilon_{\xi} \delta_{\xi\xi})(1 - \epsilon_{\eta} \delta_{\eta\eta})(1 - \epsilon_{\zeta} \delta_{\zeta\zeta}) \bar{R} = R$$

UNCONDITIONALLY STABLE IF

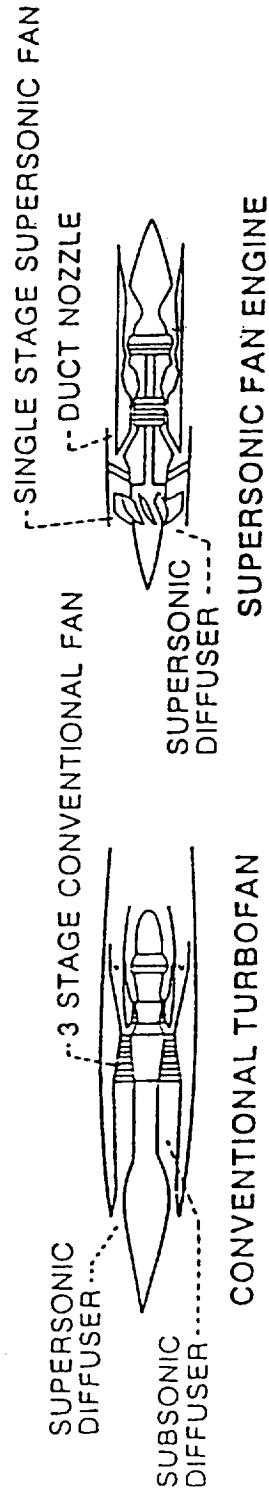
$$\epsilon \geq \frac{1}{4} \left[\left(\frac{\lambda}{\lambda^*} \right)^2 - 1 \right]$$

WHERE

λ^* IS COURANT LIMIT OF THE UNSMOOTHED SCHEME
 λ IS THE LARGER OPERATING COURANT NUMBER

SUPERSONIC FAN

(NASA LeRC ASSESSMENT STUDY)



SUPERSONIC FAN ENGINE FEATURES

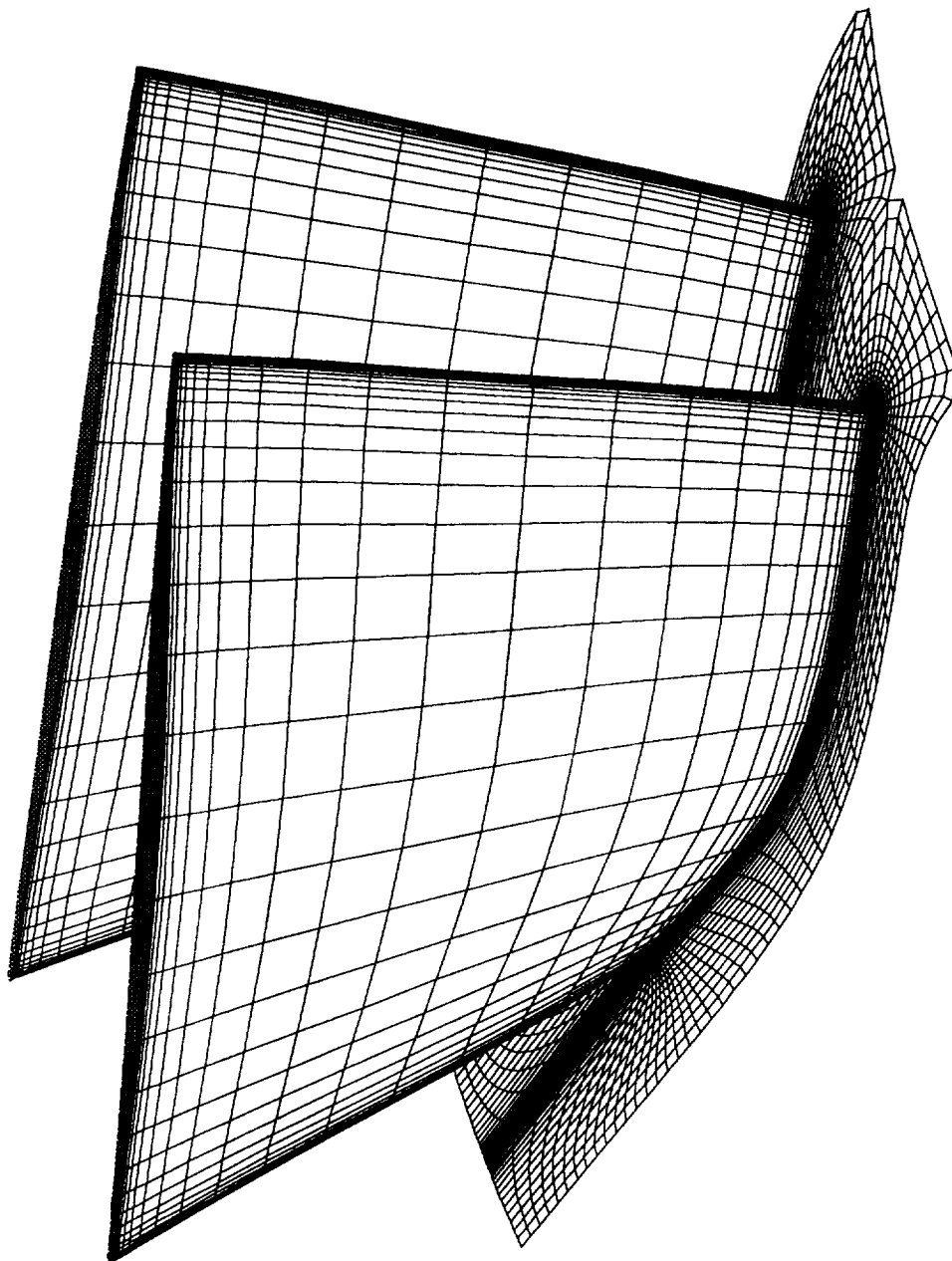
- SHORT, ALL SUPERSONIC INLET
- SINGLE STAGE SUPERSONIC FAN
- BPR DECREASES WITH M_0

IMPLICATIONS

- LOWER WEIGHT, LOWER INLET DRAG
- LOWER WEIGHT AND COST, RUGGED BLADING
- HIGHER CRUISE THRUST

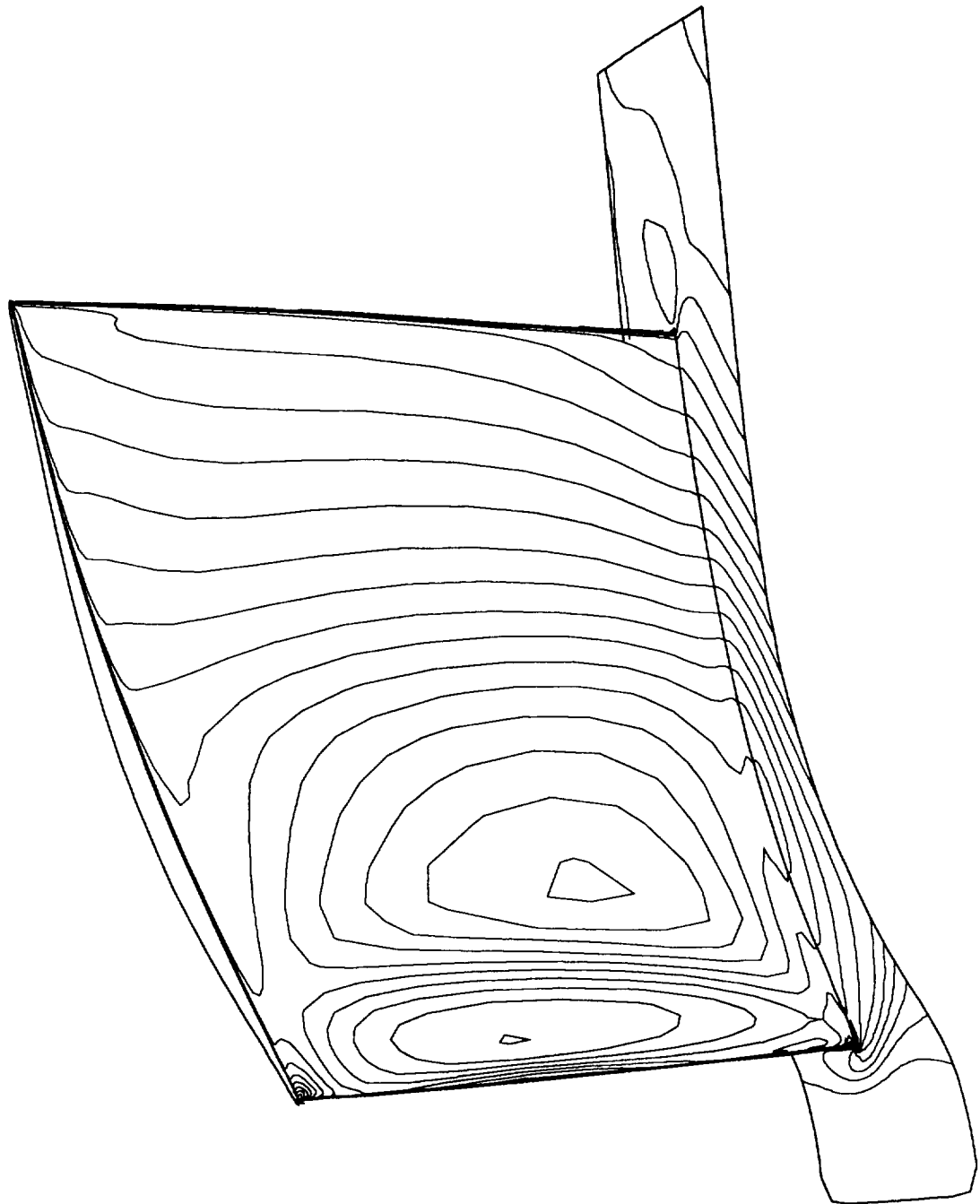
WEIGHT 15 TO 20% LOWER	CRUISE SFC-10 TO 20% LOWER
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GEOMETRY
SUPERSONIC THROUGHFLOW FAN

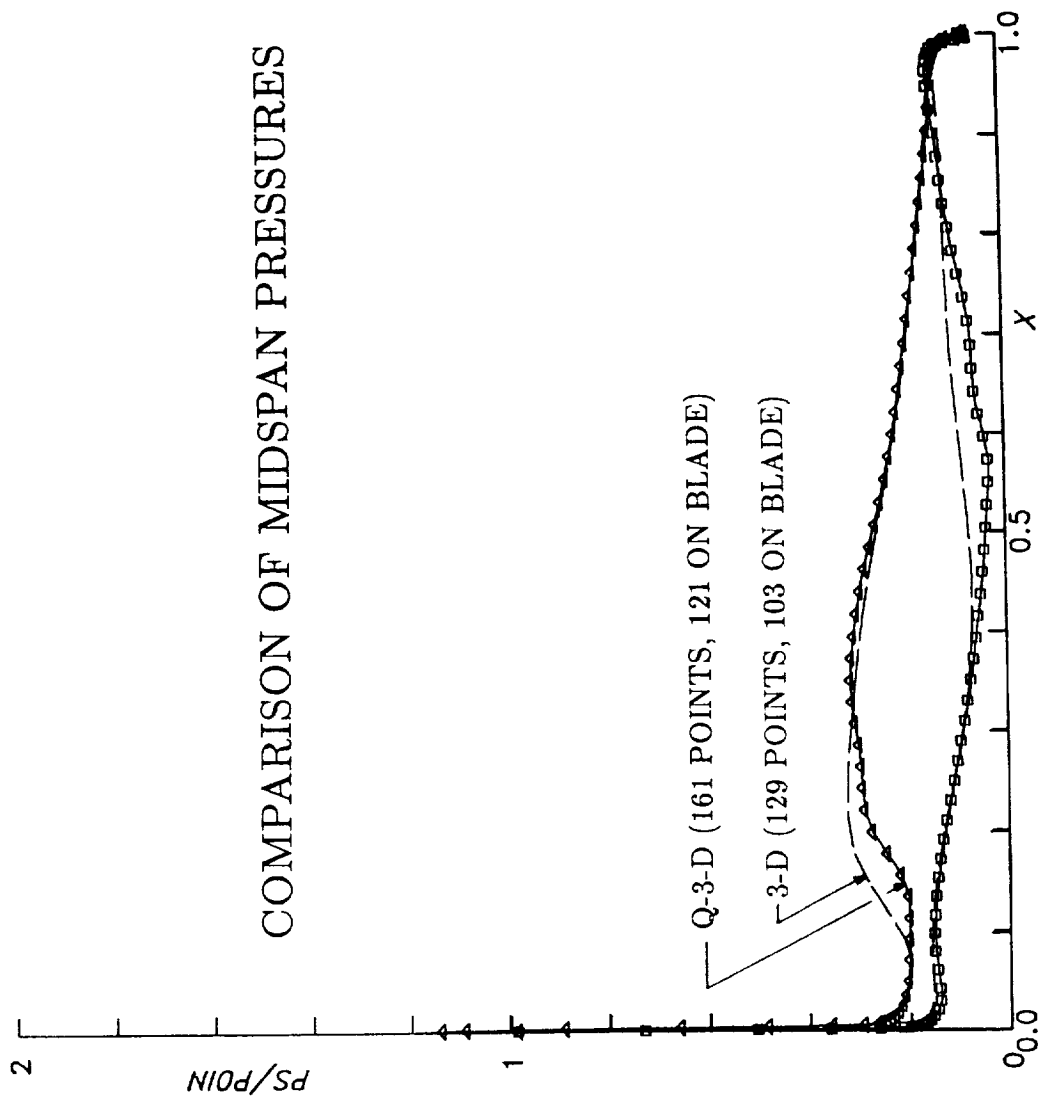


PRESSURE

SUPERSONIC THROUGHFLOW FAN, $M_{rel} = 2.5$



COMPARISON OF MIDSPAN PRESSURES

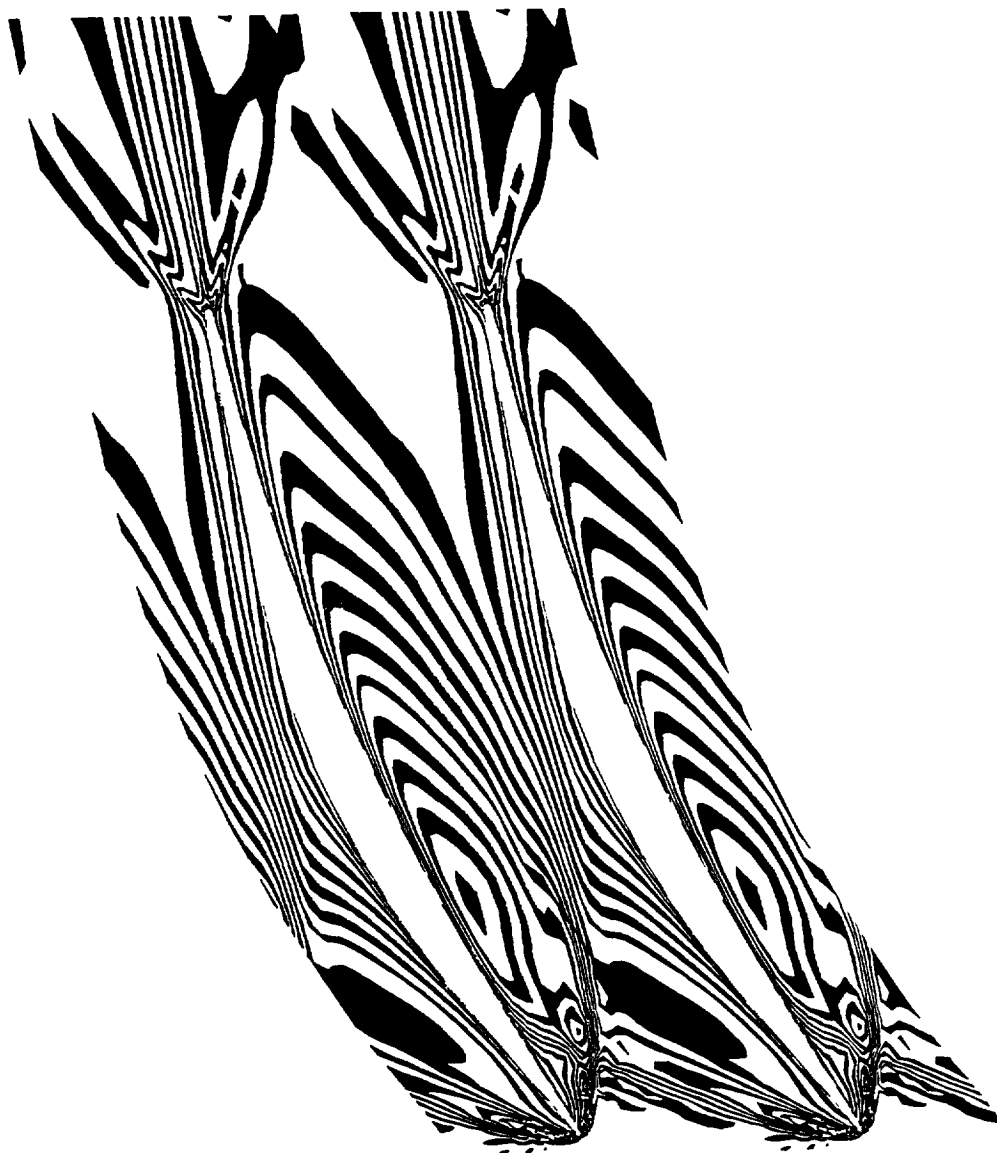


SUPERSONIC THROUGHFLOW FAN 3-D $K=17$

MACH 2.000 RE 554500. ALPHA 0.00 ITER 1000

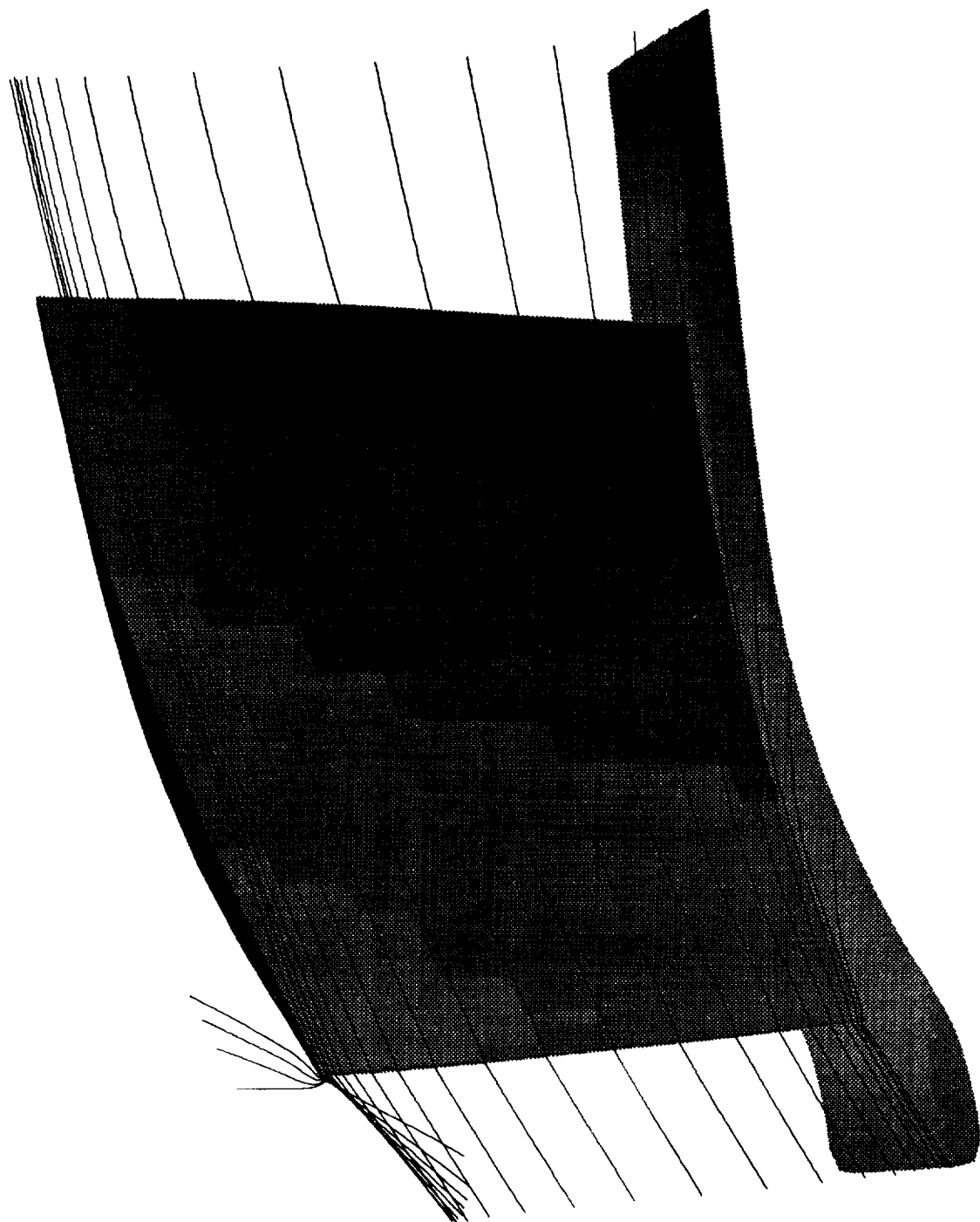
DENSITY

SUPERSONIC THROUGHFLOW FAN, $M_{rel} = 2.5$

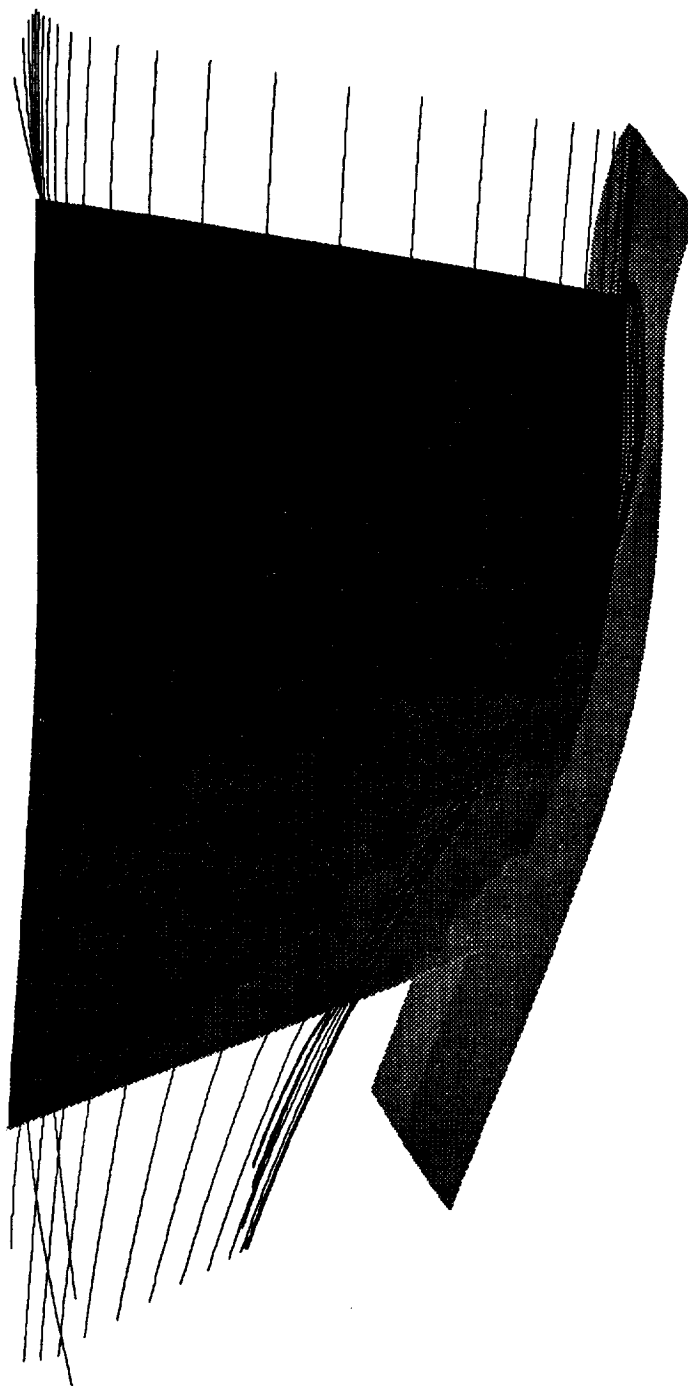


PARTICLE TRACES

SUPERSONIC THROUGHFLOW FAN, $Ma_1 = 2.5$



PARTICLE TRACES



SUMMARY

PHYSICS

- 3-D NAVIER-STOKES ANALYSIS FOR STEADY INTERNAL FLOWS
- CARTESIAN FORMULATION ROTATING ABOUT X-AXIS
- THIN LAYER IN STREAMWISE DIRECTION, FULL N-S IN OTHERS
- BALDWIN-LOMAX TURBULENCE MODEL

NUMERICS

- FINITE-DIFFERENCE FORM ON GENERAL BODY FITTED GRID
- EXPLICIT MULTISTAGE RUNGE-KUTTA SCHEME
- VARIABLE $\Delta t_{i,j}$ & IMPLICIT RESIDUAL SMOOTHING

RESULTS

- SUPERSONIC THROUGHFLOW FAN
- HORSESHOE VORTEX AHEAD OF CYLINDER
- ANNULAR TURBINE CASCADE

FUTURE

- NEW FAN DESIGN WITH CONVERGING HUB, HIGH TURNING
- FINER GRIDS ON NAS
- TIP CLEARANCE
- MULTIGRID